1. Place the elements {1, 2, 3, 4, 5} into a
    a) row vector $\underline{a}$
    b) column vector $\underline{b}$

2. What are some other notations for vectors $\underline{a}$ and $\underline{b}$?

3. What are the dimensions of vectors $\underline{a}$ and $\underline{b}$?

4. What are $\underline{a}^T$ and $\underline{b}^T$ (T = transposed vectors)?

5. Freehand the following vectors. What is the magnitude (r) and direction (θ, the angle counterclockwise from the x-axis)?

a) [1, -1] [-1, 1] [-1, -1] [1, 0] [0, 1]

b) [1, 1] $[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}]$ [10, 10] $[5\sqrt{2}, 5\sqrt{2}]$

6. Add the following vectors. Plot the three vectors.

[1, 1] + [1, -1] =

[-1, -1] + [1, -1] =

[1, 0] + [0, 1] =

[1, 1] + [1, 1] =

7 a) A ship captain logs the northward and eastward movement of a ship for 5 hours. Plot the course.

hour	N	E
1	10	10
2	20	20
3	30	10
4	30	10
5	10	10

b) An airplane flies due north for 5 hours, with wind coming from the west (moving east). Plot the course.

hour	airplane speed	wind speed
1	100	50
2	200	100
3	300	50
4	300	50
5	100	50

c) What is the resultant vector $\vec{x}$? Plot it.

$$\vec{x} = \overrightarrow{x_1} + \overrightarrow{x_2} + \overrightarrow{x_3} + \overrightarrow{x_4} + \overrightarrow{x_5}$$

i) $\overrightarrow{x_1}$ = [1, 0]
$\overrightarrow{x_2}$ = [2, 0]
$\overrightarrow{x_3}$ = [3, 0]
$\overrightarrow{x_4}$ = [4, 0]
$\overrightarrow{x_5}$ = [5, 0]

ii) $\overrightarrow{x_1}$ = [10, 10]
$\overrightarrow{x_2}$ = [20, 20]
$\overrightarrow{x_3}$ = [10, 30]
$\overrightarrow{x_4}$ = [10, 30]
$\overrightarrow{x_5}$ = [10, 10]

iii) $\overrightarrow{x_1}$ = [50, 100]
$\overrightarrow{x_2}$ = [100, 200]
$\overrightarrow{x_3}$ = [50, 300]
$\overrightarrow{x_4}$ = [50, 300]
$\overrightarrow{x_5}$ = [50, 100]

8. Add the following terms. Plot.

$\underline{x}$ = [1, 1] $\underline{y}$ = [0, 1] $\underline{z}$ = [1, 0]

a) $5\underline{x} + 2\underline{y}$

b) $5\underline{x} + 2\underline{z}$

c) $\underline{x} + 2\underline{y} + 3\underline{z}$

d) $3\underline{x} + 2\underline{y} + \underline{z}$

9. Calculate the unit vector $\vec{u} = \frac{\vec{x}}{\|\vec{x}\|}$ and plot.

a) $\vec{x}$ = [1, 1] b) $\vec{x}$ = [10, 0] c) $\vec{x}$ = [10, 10]

10. Calculate the dot product (inner product) between the vectors. Are they orthogonal (dot product = 0)?

$[\ 1,\ \ 2] \cdot [3,\ \ 4]^T\ =$

$[-1, -1] \cdot [1, -1]^T =$

$[\ 1,\ \ 0] \cdot [0,\ \ 1]^T\ =$

$[\ 1,\ \ 1] \cdot [1,\ \ 1]^T\ =$

11. a) What are the dimensions of $\underline{a}$ and $\underline{b}$? Which is the row vector and which is the column vector?

$$\underline{a} = [1, 2, 3, 4, 5] \quad \text{and} \quad \underline{b} = [1, 2, 3, 4\ 5]^T$$

b) Matrix multiplication (the inner product, or dot product) only works if the inner dimensions match. Do it if you can.

$\underline{a}\,\underline{b} =$

$\underline{a}^T\underline{b} =$

$\underline{a}\,\underline{b}^T =$

$\underline{b}\,\underline{a} =$

$\underline{b}^T\underline{a} =$

$\underline{b}\,\underline{a}^T =$

12. Can you "imagine" that *i* squared is negative one?

13. Is each number: real, imaginary, or complex?

a) 22
b) $2i$
c) $22 + 2i$
d) $22 + 33$
e) $22 + 33i$
f) $22i - 33$
g) 0
h) $\frac{\sqrt{2}}{2}$
i) $\frac{\sqrt{2}}{2} - \sqrt{2}i$
j) 0
k) π
l) $\sqrt{2}$

14. Simplify by using imaginary numbers

a) $\sqrt{-1}$ b) $\sqrt{-4}$ c) $\sqrt{-9}$ d) $\sqrt{-16}$

e) $\sqrt{-8}$ f) $\sqrt{-18}$ g) $\sqrt{-50}$ h) $\sqrt{-2}$

15. Simplify by using complex numbers

a) $\frac{2+\sqrt{-4}}{2}$ b) $\frac{2-\sqrt{-4}}{2}$ c) $\frac{2\pm\sqrt{-4}}{2}$

d) $\frac{0+\sqrt{-9}}{2}$ e) $\frac{2-\sqrt{-8}}{2}$ f) $\frac{2\pm\sqrt{-9}}{2}$

16. Combine like terms to simplify the complex numbers

a) $1 + 2 + 3 + 4i + 5i + 6i + 7i =$

b) $(1 + 2 + 3) + (4i + 5i + 6i + 7i) =$

c) $(1 + 2 + 3 + 4i + 5i) + (6i + 7i) =$

d) $(1 + 2) + 3 + (4i + 5i + 6i + 7i) =$

e) $7i + 6i + 5i + 4i + 3 + 2 + 1 =$

f) $(7i + 6i + 5i + 4i) + 3 + 2 + 1 =$

g) $4i + 1 + 2 + 7i + 3 + 5i + 6i =$

17. Multiply. Recall that $i^2 = -1$.

a) $2i \times 3 =$

b) $-2 \times 3i =$

c) $2i \times -3i =$

d) $-2i \times -3i =$

18. Simplify.

a) i^2 =

b) i^3 =

c) i^4 =

d) i^5 =

e) i^6 =

f) $-2i \times -2i =$

g) $-2i \times -2i \times -2i =$

h) $-2i \times -2i \times -2i \times -2i =$

i) $-2i \times -2i \times -2i \times -2i \times -2i =$

j) $(-2i)^6$

k) $(-2i)^7$

l) $(-2i)^8$

19. Distribute. Simplify.

$(1 + 2i)(3 + 4i) =$

$(1 - 2i)(3 + 4i) =$

$(1 + 2i)(3 - 4i) =$

$(1 - 2i)(3 - 4i) =$

$(2 + 3i)(4 + 5i) =$

$(2 - 3i)(4 + 5i) =$

$(2 + 3i)(4 - 5i) =$

$(2 - 3i)(4 - 5i) =$

20. Plot the parabola and find the zeros (if you can). Solve for unknown *x* using the quadratic formula. Plot the complex roots.

$$ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x^2 + 4x + 8 = 0$

$x^2 + 6x + 13 = 0$

$x^2 + 2x + 2 = 0$

$2x^2 + 4x + 4 = 0$

$x^2 + 2x + 3 = 0$

$x^2 + x + 1 = 0$

21. a) What's the dot product $\vec{x}\vec{x}^T$ for k = 1, 2, 3, 4, and 5?

$$\vec{x} = [1, 2, 3, 4, \ldots, k]$$

b) What happens to $\vec{x}\vec{x}^T$ as k becomes large?

22. a) What's the dot product $\vec{x}\vec{x}^T$ for k = 1, 2, 3, 4, and 5?

$$\vec{x} = [\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{k}]$$

b) True or False (Hint: look up the Basel Problem):

As *k* becomes large (more than 1,000),

$\vec{x}\vec{x}^T$ converges to (approximately equals) $\frac{\pi^2}{6}$