1. Calculate the values for the exponential function $f(x) = 10^x$ and show how many zeros are in the number. Freehand in the space to the right.

x	*f(x)*	*# zeros*
0		
1		
2		
3		
4		
5		
6		

2. How many commas are in the following numbers?
   a) 1,000 ______
   b) 1,000,000 ______
   c) 1,000,000,000 ______
   d) 1,000,000,000,000 ______

3. What does a "six figure salary" mean? Seven figures?

4. How much would each check be worth?

   a) one followed by three zeros ____________________
   b) one followed by six zeros ____________________
   c) one followed by nine zeros ____________________

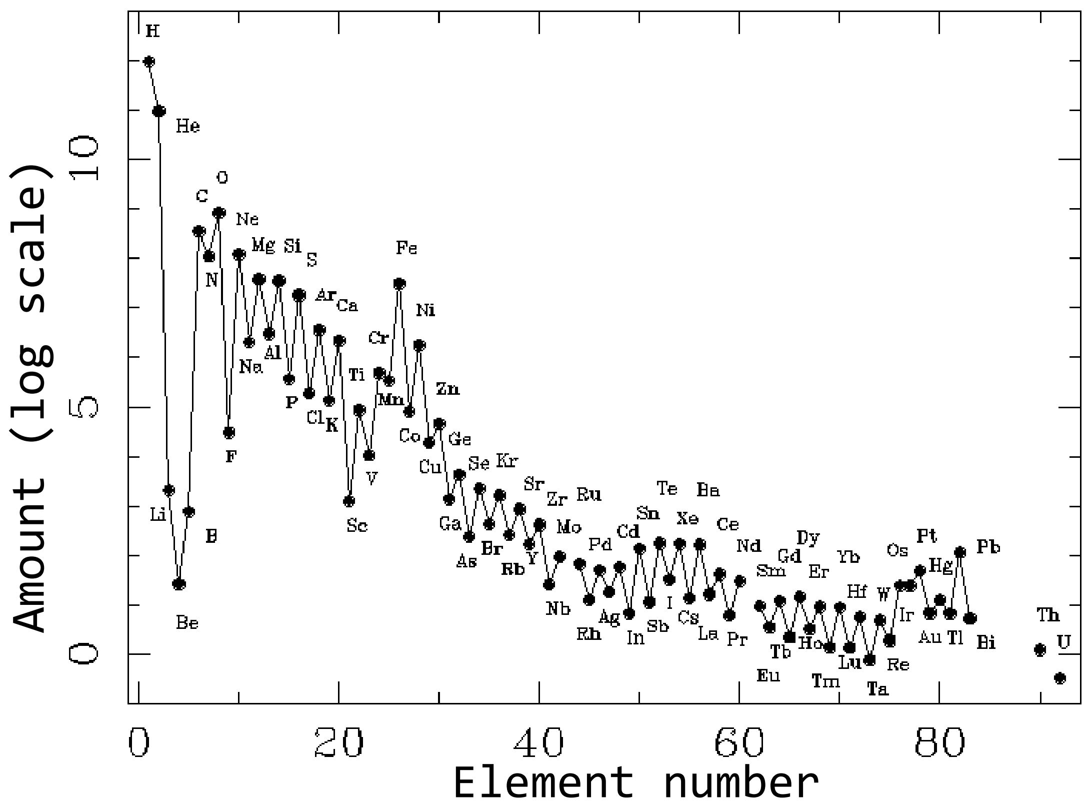
5. Ten raised to what power *x* results in the following numbers?

x	10^x
	1
	10
	100
	1,000
	10,000
	100,000
	1,000,000

6. What is the $\log_{10}(x)$? That is, ten raised to what power results in the following numbers?

x	$\log_{10}(x)$
1	_____
10	_____
100	_____
1,000	_____
10,000	_____
100,000	_____
1,000,000	_____
1,000,000,000	_____
1,000,000,000,000	_____

7. The y-axis on the below figure shows $\log_{10}(y)$, where *y* is the amount of chemical element *x* on planet Earth. That is, the y-axis is shown using a "logarithmic scale" while x is not. This is called a "semi-log" plot.



a) What is $\log_{10}(y)$ for element $x = 1$ (leftmost dot labeled H)? Round to the nearest whole number. Just read it from the plot.

b) What is $\log_{10}(y)$ for element $x = 2$ (dot labeled He)? Round to the nearest whole number. Just read it from the plot.

c) What are the actual amounts *y* for H (x=1) and He (x=2)?

d) How much more H than He is on planet Earth?

e) How much more C (x=6) than Fe (x=26)?

f) Freehand y vs. x (regular plot, <u>not</u> semi-log) on the back.

8. The Richter scale for earthquake strength is a logarithmic ($\log_{10}$) scale that goes from 0-10.

a) How much more powerful is an earthquake 10 vs 9?

b) How much more powerful is an earthquake 10 vs 8?

c) How much more powerful is an earthquake 10 vs 7?

d) How much more powerful is an earthquake 10 vs 6?

e) How much more powerful is an earthquake 10 vs 5?

f) How much more powerful is an earthquake 10 vs 4?

g) How much more powerful is an earthquake 10 vs 3?

h) How much more powerful is an earthquake 9 vs 8?

i) How much more powerful is an earthquake 9 vs 7?

j) How much more powerful is an earthquake 9 vs 6?

k) How much more powerful is an earthquake 9 vs 5?

l) How much more powerful is an earthquake 9 vs 4?

m) How much more powerful is an earthquake 8 vs 7?

n) How much more powerful is an earthquake 8 vs 6?

o) How much more powerful is an earthquake 8 vs 5?

p) How much more powerful is an earthquake 8 vs 4?

q) How much more powerful is an earthquake 7 vs 6?

r) How much more powerful is an earthquake 7.5 vs 4.5?

s) How much more powerful is an earthquake 6.5 vs 5.5?

t) How much more powerful is an earthquake 6.5 vs 5.0?

9. Use the "LOG" key on a calculator. Show 2 decimal places.

x	$f(x) = \log_{10}(x)$	x	$f(x) = \log_{10}(x)$
1	_____		
2	_____	200	_____
3	_____	300	_____
4	_____	400	_____
5	_____	500	_____
6	_____	600	_____
7	_____	700	_____
8	_____	800	_____
9	_____	900	_____
10	_____	1,000	_____
20	_____	2,000	_____
30	_____	3,000	_____
40	_____	4,000	_____
50	_____	5,000	_____
60	_____	6,000	_____
70	_____	7,000	_____
80	_____	8,000	_____
90	_____	9,000	_____
100	_____	10,000	_____

10. What's the domain and range for $f(x) = \log_{10}(x)$?

11. Use the "LOG" key on a calculator. Show 2 decimal places.

x	$f(x) = \log_{10}(x)$	x	$f(x) = \log_{10}(x)$
1.0	_____	0.010	_____
0.9	_____	0.009	_____
0.8	_____	0.008	_____
0.7	_____	0.007	_____
0.6	_____	0.006	_____
0.5	_____	0.005	_____
0.4	_____	0.004	_____
0.3	_____	0.003	_____
0.2	_____	0.002	_____
0.10	_____	0.0010	_____
0.09	_____	0.0009	_____
0.08	_____	0.0008	_____
0.07	_____	0.0007	_____
0.06	_____	0.0006	_____
0.05	_____	0.0005	_____
0.04	_____	0.0004	_____
0.03	_____	0.0003	_____
0.02	_____	0.0002	_____
0.01	_____	0.0001	_____

12. Using your answers to problems 9 and 11 (without a calculator), what is the $\log_{10}$ of:

a) 90,000
b) 900,000
c) 9,000,000
d) 0.00002
e) 0.00005
f) 0.00008

13. Use the "LOG" key on a calculator. Show 2 decimal places.

$\log_{10}(1 \times 10^2) =$

$\log_{10}(1 \times 10^3) =$

$\log_{10}(1 \times 10^4) =$

$\log_{10}(2 \times 10^2) =$

$\log_{10}(2 \times 10^3) =$

$\log_{10}(2 \times 10^4) =$

$\log_{10}(5 \times 10^2) =$

$\log_{10}(5 \times 10^3) =$

$\log_{10}(5 \times 10^4) =$

14. We define $log_b(n) = a$ such that $b^a = n$.

$log_2(1) =$

$log_3(1) =$

$log_4(1) =$

$log_2(2) =$

$log_3(3) =$

$log_4(4) =$

$log_2(4) =$

$log_3(9) =$

$log_4(16) =$

$log_2(8) =$

$log_3(27) =$

$log_4(64) =$

15. Don't use a calculator.

$log_2(1) =$ $\quad$ $log_3(1) =$ $\quad$ $log_4(1) =$

$log_2(\frac{1}{2}) =$ $\quad$ $log_3(\frac{1}{3}) =$ $\quad$ $log_4(\frac{1}{4}) =$

$log_2(\frac{1}{4}) =$ $\quad$ $log_3(\frac{1}{9}) =$ $\quad$ $log_4(\frac{1}{16}) =$

$log_2(\frac{1}{8}) =$ $\quad$ $log_3(\frac{1}{27}) =$ $\quad$ $log_4(\frac{1}{64}) =$

16. Use a computer to plot the exponential functions:

$f(x) = 10^x$ $\quad$ $f(x) = 3^x$ $\quad$ $f(x) = e^x$ $\quad$ $f(x) = 2^x$

Freehand below on the same axis. What is the approximate value of Euler's number *e*?

17. How many digits would the number have?

Procedure: step 1. Take the $\log_{10}$

step 2. Simplify using $\log_{10}(n^m) = m \log_{10}(n)$

step 3. Use a calculator, round up.

How many digits in 2^3 ?

How many digits in 2^{10} ?

How many digits in 2^{100} ?

How many digits in 2^{4321} ?

18. Calculate pH, a logarithmic measure of how acidic a substance is.

x = actual (absolute) amount of acidity

$pH = -\log_{10}(x)$

x	pH = $-\log_{10}(x)$	*x*	pH = $-\log_{10}(x)$
0.1	_____	1×10^{-7}	_____
0.01	_____	1×10^{-8}	_____
0.001	_____	1×10^{-14}	_____
0.0001	_____	1	_____
0.00001	_____	10	_____

19. The amount of acidity x is an exponential function of pH

$$x = 10^{-pH}$$

x	pH = $-\log_{10}(x)$	*x*	pH = $-\log_{10}(x)$
__________	1	__________	7
__________	2	__________	8
__________	3	__________	14
__________	4	__________	0
__________	5	__________	-1

20. a) Which is more acidic (larger x): pH 1 or pH 2?

b) How much more acidic is pH 1 vs pH 2?

c) How much more acidic is pH 2 vs pH 3?

d) How much more acidic is pH 1 vs pH 3?

e) How much acidity (x) is present in neutral water with pH = 7?

f) How much more acidic is pH 1 vs neutral water (pH = 7)?

g) How much more acidic is vinegar (pH 3.5) vs water (pH = 7)?

h) Can pH be zero or even negative? Give an example.

21. Calculate the value *A* of an investment based on a principal invested *P* = $1000 using the formulas for periodically compounded interest and continuously compounded interest. In both formulas, *r* is the interest rate as a decimal (e.g. 8% interest means 0.08) and t is the number of years that the interest accrues.

<u>periodically compounded</u>

$$A(t) = P(1 + \frac{r}{n})^{nt}$$

n = number of times interest is compounded per year

<u>continuously compounded</u>

$$A(t) = Pe^{rt}$$

a) Use a computer to plot *A(t)* vs. *t* on the same axis. Freehand.
- *r* = 8% interest
- domain 0 < *t* < 40 years
- 4 curves for interest compounded annually, quarterly, monthly, daily, and continuously

b) What is the value of each investment (*A*) after 40 years?

compounded annually:

compounded quarterly:

compounded monthly:

compounded daily:

compounded continuously:

22. a) The "for sale price" of a house is \$250,000. A bank offers a mortgage, a contract in which you pay the monthly amount M over t years at a "nice" annual interest rate of 3.5%. How much would you pay total *(12Mt)* over a 40 year contract? 30 years? 10 years? (Let n = 12.)

$$M = \frac{P(\frac{r}{n})}{1 - (1 + \frac{r}{n})^{-nt}}$$

b) The inflation rate r is 3.5% per year, meaning money itself becomes 3.5% less valuable every year. This is a reason why prices increase over time. Fresh squeezed orange juice is \$5 today. How much in 50 years?

$$A(t) = P(1 + r)^t$$

23. Cells grow and divide continuously at an exponential rate in a petri dish. The hourly growth rate r is a decimal, the initial amount of cells is N_0, and here t is in hours.

$$N(t) = N_0 \, e^{rt}$$

a) Plot the following curves on the domain $0 < t < 72$ hours

i) N_0 = 1 million cells, r = 10% per hour growth rate

ii) N_0 = 10 million cells, r = 1% per hour growth rate

iii) N_0 = 5 million cells, r = 5% per hour growth rate

b) How many cells after 24, 48, and 72 hours for each petri dish?

24. Radioactive substances undergo "decay" in which the matter decomposes. Exactly half of the material decomposes during a fixed period of time known as the half-life, denoted τ (tau, the Greek letter t). *A(t)* is the amount remaining of an initial amount A_0 after *t* years

$$A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{t}{\tau}}$$

Uranium has a half life of 4.5 billion years, and there are about 40 trillion tons buried deep inside the Earth.

years elapsed (billions)	half lives elapsed	uranium remaining (trillions of tons)
0	__________	__________
4.5	__________	__________
9	__________	__________
18	__________	__________
36	__________	__________
40	__________	__________
50	__________	__________
100	__________	__________

25. Carbon-14 has a half life τ of 5,730 years. Living things have A_0 = 100% (maximum level) of carbon-14, and when they die the carbon-14 level decreases exponentially. The age of a fossil t is calculated from A = the percent of carbon-14 reminaing:

$$t = \frac{\tau}{-0.693} \ln\left(\frac{A}{A_0}\right)$$

% carbon-14 remaining	fraction $\frac{A}{A_0}$	fossil age (years)
100%	__________	__________
90%	__________	__________
30%	__________	__________
3%	__________	__________
0.3%	__________	__________